

MODELING NON-STATIONARITIES IN HIGH-FREQUENCY FINANCIAL TIME SERIES*

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We study tick-by-tick financial returns belonging to the FTSE MIB index of the Italian Stock Exchange (Borsa Italiana). We find that non-stationarities detected in other markets in the past are still there. Moreover, scaling properties reported in the previous literature for other high-frequency financial data are approximately valid as well. Finally, we propose a simple method for describing non-stationary returns, based on a non-homogeneous normal compound Poisson process and we test this model against the empirical findings. It turns out that the model can reproduce several stylized facts of high-frequency financial time series.

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1. Introduction. The recent rise in the availability of high-frequency financial data has seen an increase in the number of studies focusing on the areas of classification and modeling of financial markets at the ultra-high frequency level. The development of models that are able to reflect the various observed phenomena of real data is an important step towards obtaining a full understanding of the fundamental stochastic processes driving the market. The statistical properties of high-frequency financial data and market micro-structural properties were studied by means of different tools, including phenomenological models of price dynamics and agent-based market simulations (see [Goodhart and O'Hara \(1997\)](#), [O'Hara \(1999\)](#), [Madhavan \(2000\)](#), [Scalas, Gorenflo and Mainardi \(2000\)](#), [Mainardi et al. \(2000\)](#), [Dacorogna et al. \(2001\)](#), [Raberto et al. \(2001\)](#), [Cincotti et al. \(2003\)](#), [Luckock \(2003\)](#), [Scalas et al. \(2004\)](#), [Pastore, Ponta and Cincotti \(2010\)](#), [Ponta, Pastore and Cincotti \(2011\)](#), [Ponta, Raberto and Cincotti \(2011\)](#), [Mandelbrot \(1963\)](#), [Mandelbrot \(1997\)](#), [Müller et al. \(1990\)](#), [Mantegna and Stanley \(1995\)](#), [Gopikrishnan et al. \(2000\)](#), [Hautsch \(2012\)](#)).

Various studies on high-frequency econometrics appeared in the literature using the autoregressive conditional duration models (see [Engle and Russell \(1997\)](#), [Engle and Russell \(1998\)](#), [Bauwens and Giot \(2000\)](#), [Lo, MacKinlay and Zhang \(2002\)](#)). Alternative stochastic models were also proposed, e.g., diffusive models, ARCH-GARCH models, stochastic volatility models, models based on fractional processes, models based on subordinate processes (see [Cont and Bouchaud \(2000\)](#), [Chowdhury and Stauffer \(1999\)](#), [Hardle and Kirman \(1995\)](#), [M. Levy and Solomon \(1995\)](#), [Lux and Marchesi \(1999\)](#), [Stauffer and Sornette \(1999\)](#), [Youssefmir and Huberman \(1997\)](#)). An important variable is the order imbalance. Many existing studies analyze order imbalances around specific events or over short periods of time. For example in [Blume, Mackinlay and Terker \(1989\)](#) order imbalances are analyzed around the October 1987 crash. [Chan and Fong \(2000\)](#) analyze how order imbalances change the contemporaneous relation between stock volatility and volume using data for about six months. A large

body of research examines the effect of the bid-ask spread and the order impact on the short-run behavior of prices (see [Stoll and Whaley \(1990\)](#), [Hauser and Lauterbach \(2003\)](#), [Chordia, Roll and Subrahmanyam \(2002\)](#), [Ponzi, Lillo and Mantegna \(2009\)](#), [Svorenecik and Slanina \(2007\)](#), [Wyart, Bouchaud and et al. \(2008\)](#), [Moro et al. \(2009\)](#), [Perelló et al. \(2008\)](#), [Preis, Schneider and Stanley \(2011\)](#), [Kumaresan and Krejic \(2010\)](#), [Zaccaria et al. \(2010\)](#), [Lim and Coggins \(2005\)](#), [Weber and Rosenow \(2005\)](#), [Bouchaud \(2005\)](#)). Other studies have examined trading activity as measured by the average number of trades in a unit time (see [Bonanno, Lillo and Mantegna \(2000\)](#), [Plerou et al. \(2000\)](#)). However, aggregation into uniform time intervals may affect the analysis, since choosing a short unit time interval may result in many points with none or very few trades, artificially altering the heteroskedasticity of the process, while using a long unit time interval averages out multiple transactions, and the fine timing structure of the data can be lost (see [Engle and Russell \(1998\)](#)).

For this reason, another important empirical variable is the waiting time between two consecutive transactions (see [Scalas et al. \(2004\)](#), [Scalas \(2006a\)](#), [Scalas \(2006b\)](#)). Empirically, in the market, during a trading day the activity is not constant (see [Engle and Russell \(1997\)](#), [Engle and Russell \(1998\)](#)) leading to fractal-time behavior (see [Hudson and Mandelbrot \(2010\)](#), [Vrobel \(2011\)](#)). Indeed, as a consequence of the double auction mechanism, waiting times between two subsequent trades are themselves stochastic variables (see [Scalas \(2006a\)](#), [Scalas \(2006b\)](#), [Scalas \(2007\)](#), [Politi and Scalas \(2008\)](#)). They may also be correlated to returns (see [Raberto, Scalas and Mainardi \(2002\)](#)) as well as to traded volumes. In the last few years in order to investigate tick-by-tick financial time series, the continuous-time random walk (CTRW) was used (see [Scalas, Gorenflo and Mainardi \(2000\)](#), [Masoliver, Montero and Weiss \(2003\)](#), [Ivanov et al. \(2004\)](#), [Scalas \(2006a\)](#), [Scalas \(2006b\)](#)). It turned out that interorder and intertrade waiting-times are not exponentially distributed. Therefore, the jump process of tick-by-tick prices is non-Markovian (see

Scalas, Gorenflo and Mainardi (2000), Scalas (2006a), Scalas (2006b)). Bianco and Grigolini applied a new method to verify whether the intertrade waiting time process is a genuine renewal process (see Goldstein, Morris and Yen (2004), Embrechts, Klüppelberg and Mikosch (1997), Press, Flannery and Teukolsky (1992)). This was assumed by the CTRW hypothesis in Scalas, Gorenflo and Mainardi (2000). They found that intertrade waiting-times do follow a renewal process. Indeed, trading via the order book is asynchronous and a transaction occurs only if a trader issues a market order. For liquid stocks, waiting times can vary in a range between fractions of a second to a few minutes, depending on the specific stock and on the market considered. In Raberto, Scalas and Mainardi (2002), the reader can find a study on General Electric stocks traded in October 1999. Waiting times between consecutive prices exhibit 1-day periodicity, typical of variable intraday market activity. Moreover, the survival probability (the complementary cumulative distribution function) of waiting times is not exponentially distributed (see Mainardi, Gorenflo and Scalas (2004), Scalas (2006a)), but is well fitted by a Weibull function (see Engle and Russell (1994), Engle and Russell (1997), Engle and Russell (1998), Raberto, Scalas and Mainardi (2002), Ponta et al. (2012)).

Here, inspired by Bertram (2005), we propose a model based on non-homogeneous Poisson processes. The paper is organized as follows. Section 2 describes the dataset used considering a general description in Subsection 2.1, and the FTSE MIB index in Subsection 2.2. Section 3 (and in particular Subsections 3.1 and 3.2) describes the statistical analysis of the single assets and of the FTSE MIB index, respectively as well as the scaling analysis; Section 4 contains the bivariate analysis whereas Section 5 is devoted to the model and the results. Finally Section 6 presents the conclusions of this work.

2. Data set.

2.1. *General description.* The data set includes high-frequency trades registered at Italian Stock Exchange (BIIt or Borsa Italiana), from the 03rd of February 2011 to the 09th of March 2011. The data of February 14th 2011 are not used because, on that day, there were technical problems at BIIt. Moreover, we have removed the data of the 21st of February, as well. In fact, in that day, there was a crash in the Italian market related to the events in Lybia. We consider the 40 shares in the FTSE MIB index at the time, namely: A2A, STS, ATL, AGL, AZM, BP, BMP, PMI, BUL, BZU, CPR, DIA, ENE, EGP, ENI, EXO, F, FI, FNC, FSA, G, IPG, ISP, LTO, LUX, MS, MB, MED, PLT, PC, PRY, SPM, SRG, STM, TIT, TEN, TRN, TOD, UBI, UCG. Further information on the database and the full meaning of the symbols is available from www.borsaitaliana.it. However, Table 1 shows the meaning of the acronyms as well as the number of observations for each share. The forty stocks composing the FTSE MIB vary in their average market capitalization and exhibit different levels of trading activity with different numbers of trades over this period as summarized in the last column in Table 1 where the total number of observations in the chosen month is given. Choosing one month of high-frequency data was a trade-off between the necessity of managing enough data for significant statistical analysis and, on the other hand, the goal of minimizing the effect of external economic fluctuations leading to non-stationarities of the kind discussed by [Livan, Inoue and Scalas \(2012\)](#). For every stock, the data set consists of prices $p(t_i)$, volumes $v(t_i)$ and times of execution t_i , where i is the trade index, varying from 1 to the total number of daily trades N . These data were filtered in order to remove misprints in prices and times of execution. In particular, concerning prices, when there are multiple prices for the same time of execution, we consider only one transaction at that time and a price equal to the average of the multiple prices, and concerning the waiting times, τ , between two executions, we remove observations larger than 200 s.

2.2. *FTSE MIB Index.* The FTSE MIB Index (see [FTSEMIB \(2011\)](#)) is the primary benchmark index for the Italian equity markets. Capturing approximately 80% of the domestic market capitalisation, the Index is made up of highly liquid, leading companies across Industry Classification Benchmark (ICB) sectors in Italy. The FTSE MIB Index measures the performance of 40 shares listed on Borsa Italiana and seeks to replicate the broad sector weights of the Italian stock market. The Index is derived from the universe of stocks trading on BIt. The Index replaces the previous S&P/MIB Index, as a benchmark Index for Exchange Traded Funds (ETFs), and for tracking large capitalisation stocks in the Italian market. FTSE MIB Index is calculated on a real-time basis in EUR. The official opening and closing hours of the FTSE MIB Index series coincide with those of BIt markets and are 09:01 and 17:31 respectively. The FTSE MIB Index is calculated and published on all days when BIt is open for trading.

FTSE is responsible for the operation of the FTSE MIB Index. FTSE maintains records of the market capitalisation of all constituents and other shares and makes changes to the constituents and their weightings in accordance with the Ground Rules. FTSE carries out reviews and implement the resulting constituent changes as required by the Ground Rules. The FTSE MIB Index constituent shares are selected after analysis of the Italian equity universe, to ensure the Index best represents the Italian equity markets.

The FTSE MIB Index is calculated using a base-weighted aggregate methodology. This means the level of an Index reflects the total float-adjusted market value of all of the constituent stocks relative to a particular base period. The total market value of a company is determined by multiplying the price of its stock by the number of shares in issue (net of treasury shares) after float adjustment. An Indexed number is used to represent the result of this calculation in order to make the value easier to work with and track over time. As mentioned above, the Index is computed in real time. The details on how to compute it can be found in [FTSEMIB \(2011\)](#).

3. Descriptive Univariate Unconditional Statistics. In this section, we separately consider the descriptive univariate unconditional statistics for both the forty assets and for the FTSE MIB Index. By *univariate*, we mean that, here, we do not consider correlations between the variables under study. By *unconditional*, we mean that, here, we do not consider the non-stationary and seasonal behavior of the variables under study and the possible memory effects. Correlation and non-stationarity will be discussed in the next Section.

3.1. Single Assets. In order to characterize market dynamics on a trade-by-trade level, we consider three variables: the series of time intervals between consecutive trades, τ , the trade volumes, v , and the trade-by-trade logarithmic returns, r . If $p(t_i)$ represents the price of a stock at time t_i where t_i is the epoch of the i -th trade, then we define the return as:

$$(3.1) \quad r_i = \log \frac{p(t_{i+1})}{p(t_i)}.$$

Note that $\tau = t_{i+1} - t_i$ is a random intertrade duration (and not a fixed time interval).

Among the empirical studies on τ , we mention [Golia \(2001\)](#); [Raberto, Scalas and Mainardi \(2002\)](#), concerning contemporary shares traded over a period of a few months, a study on rarely traded nineteenth century shares by [Sabatelli et al. \(2002\)](#), and results on foreign exchange transactions by [Takayasu \(2002\)](#) and [Marinelli, Rachev and Roll \(2001\)](#).

Tables [2](#), [3](#) and [4](#) contain the descriptive statistics, evaluated for the entire sample, for the time series $\tau_i^h = t_{i+1}^h - t_i^h$ (with $t_0^h = 0$), v_i^h and r_i^h , where the superscript h denotes the specific share.

In Table [2](#) the third and fourth columns give the two parameters of a Weibull distribution fit. The Weibull distribution has the following survival function:

$$(3.2) \quad \mathbb{P}(\tau > t) = P(t|\alpha, \beta) = \exp(-\alpha t^\beta),$$

where β is the shape parameter and α is the scale parameter. The values given in Table 2 were fitted using the moment method described in [Politi and Scalas \(2008\)](#). The quality of these fits is pictorially shown in Fig. 1 for A2A, EXO, MS and TIT, respectively. The solid line represents our Weibull fit and the circles are the empirical data. Since different companies have different average intertrade duration $\langle \tau^h \rangle$ (see the second column in Table 2), they are also characterized by a different scale parameter α whereas the shape parameter β is almost the same for all the forty time series. Following [Ivanov et al. \(2004\)](#), a scaling function $P(t|\beta^*)$ can be defined:

$$(3.3) \quad P(t|\beta^*) = \exp \left(-(t/\langle \tau \rangle)^{\beta^*} \right)$$

where $\beta^* = \langle \beta \rangle = 0.78$. To test the hypothesis that there is a universal structure in the intertrade time dynamics of different companies, we rescale the survival functions plotting them against $t/\langle \tau^h \rangle$. We find that, for all companies, data approximately conform to a single scaled plot given by (3.3) as shown in Fig. 2 (see also [Stauffer and Stanley \(1995\)](#); [Ivanov et al. \(2004\)](#); [Politi and Scalas \(2008\)](#)). Such a behavior is a hallmark of scaling, and is typical of a wide class of physical systems with universal scaling properties [Bunde and Havlin \(1994\)](#). Even if [Eisler and Kertész \(2006\)](#) showed that the scaling (3.3) is far from being universal, at least for the New York Stock Exchange, it is remarkable to find it again for a different index in a different market and seven years later with respect to the findings of [Ivanov et al. \(2004\)](#). The goodness of fits is given in the last column of Table 2, where we report the Anderson-Darling statistics for the transformed random variable $z_\tau^h = \alpha \tau^\beta$. z_τ should follow an exponential distribution with parameter $\mu = 1$, if τ is distributed according to a Weibull distribution. A glance at Fig. 3 immediately shows that this is not the case; for $z_\tau > 4$ there are significant deviations from the exponential law, whereas this is approximately satisfied for $z_\tau \leq 4$. This fact is reflected by the high values of the AD statistics for

which the critical value at 0.05 significance level is 1.34. In other words, the Weibull null hypothesis is rejected for all the time series.

In this paper, we do not study volumes v^h , but we have presented their descriptive statistics in Table 3 for the sake of completeness.

The descriptive statistics for trade-by-trade returns r^h can be found in Table 4. Notice that there is excess kurtosis. Figure 4 shows the histogram for the prices assets A2A, EXO, MS and TIT, respectively in order to show how the returns are distributed. It is possible to appreciate the discrete character of returns even after the logarithmic transformation.

3.2. FTSE MIB index. As well as the single assets, we investigate the FTSE MIB index. Tables 2 and 4 summarize also the descriptive statistics of the time series τ_i^I and r_i^I respectively evaluated for the FTSE MIB index as trade-by-trade volumes are not available.

In Fig. 5 we show the survival function for the intertrade waiting time of the FTSE MIB index. The solid line represent the Weibull fit, whereas the circle represent the empirical data. The shape of the two curves is very different. Therefore, we can immediately see that intertrade times are not Weibull distributed, and, in this case, the fit does not work even as a first approximation. For this reason we have not reported values of α , β , and the Anderson-Darling statistic. Indeed, for the FTSE MIB index, the standard deviation of intertrade durations is smaller than the average intertrade duration.

Contrary to the case of single asset returns, the excess kurtosis for the FTSE MIB index is quite large. Fig. 6 shows the histogram of the returns for a bin size of 1×10^{-5} . Following Mantegna and Stanley (1995), we test the scaling of the empirical returns. As shown in Table 1, the dataset consists of 405560 records for the FTSE MIB index during the period studied (from the 03rd of February 2011 to the 09th of March 2011). From this database,

we compute the new random variable $r^I(t; \Delta t)$ defined as:

$$(3.4) \quad r^I(t; \Delta t) = \log \frac{p^I(t + \Delta t)}{p^I(t)},$$

where $p^I(t)$ is the value of the index at time t . In this way we sample returns on equally spaced and non-overlapping intervals of width Δt . We further assume that the time series is stationary so that it only depends on Δt and not on t (incidentally, we shall see that this is not the case). To characterize quantitatively the experimentally observed process, we first determine the empirical probability density function $P(r^I(\Delta t))$ of index variations for different values of Δt . We select Δt equal to 3s, 5s, 10s, 30s and 300s. In Fig. 7(a) we present a semi-logarithmic plot of $P(r^I(\Delta t))$ for the five different values of Δt indicated above. These empirical distributions are roughly symmetric and converge to the normal distribution when Δt increases. We also note that the distributions are leptokurtic, that is, they have tails heavier than expected for a normal process. A determination of the parameters characterizing the distributions is difficult especially because larger values of Δt imply a smaller number of data. Again following Mantegna and Stanley (1995), we study the probability density at zero return $P(r^I(\Delta t) = 0)$ as function of Δt . This is done in Fig. 7(b), where $P(r^I(\Delta t) = 0)$ versus Δt is shown in a log-log plot. If these data were distributed according to a symmetric α -stable distribution, one would expect the following form for $P(r^I(\Delta t) = 0)$:

$$(3.5) \quad P(r^I(\Delta t) = 0) = \frac{\Gamma(1/\alpha_L)}{\pi \alpha_L (c \Delta t)^{1/\alpha_L}},$$

where $\Gamma(\cdot)$ is Euler Gamma function, $\alpha_L \in (0, 2]$ is the index of the symmetric α -stable distribution and c is a times scale parameter. The data are well fitted (in the OLS sense) by a straight line of slope $1/\hat{\alpha}_L = 0.58$ leading to an estimated exponent $\hat{\alpha}_L = 1.72$. The best method to get the values

of $P(r^I(\Delta t) = 0)$ is determining the slope of the cumulative distribution function in $r^I(\Delta t) = 0$. In Fig. 7(c), we plot the rescaled probability density function according to the following transformation:

$$(3.6) \quad r_s^I = \frac{r^I(\Delta t)}{(\Delta t)^{1/\alpha_L}}$$

and

$$(3.7) \quad P(r_s^I) = \frac{P(r^I(\Delta t))}{(\Delta t)^{-1/\alpha_L}},$$

for $\alpha_L = \hat{\alpha}_L = 1.72$. Remarkably all the five distributions approximately collapse into a single one. It is worth noting that this result shows that the scaling, found in the S&P 500 data by Mantegna and Stanley seventeen years ago, still approximately holds in a different market and in a completely different period. In this case, we do not run hypothesis test because an eye inspection of Fig. 7(c) is sufficient to conclude that the Lévy stable fit is only approximately valid.

4. Descriptive Conditional and Bivariate Statistics. Inspired by [Bertram \(2004, 2005\)](#), in order to study the time variations of the returns during a typical trading day, we use a simple technique. We divide the trading day into equally spaced and non-overlapping intervals of length δt for $\delta t = 3, 5, 10, 30, 300, 600, 900, 1200, 1500$ and 1800 s. Let K be number of intervals and N_k the number of transaction in each interval k . For each interval we evaluate the $\gamma(k)$ indicator as a measure of volatility. $\gamma(k)$ is defined as

$$(4.1) \quad \gamma(k) = \frac{1}{N_k - 1} \sum_{i=1}^{N_k-1} |r_{k,i}^I - \langle r_k^I \rangle|;$$

where $\langle r_k^I \rangle$ is the average value of returns in the time interval k . In Fig. 8(a), as an example, we plot the average value of $\gamma(k)$ over the investigated

period as a function of the interval index k for $\delta t = 300$ s. We can see that the volatility is higher in the morning, at the opening of continuous trading, and then it decreases up to midday. There is a local increase after midday and then the volatility returns to lower values to finally grow towards the end of continuous trading. The above pattern can be reinforced by the presence of the many *day traders* whose practice is to close all their positions at the end of each trading day and reopen them in the following morning. The rationale of day traders is to avoid overnight exposure to risk. Interestingly this plot also provides us with a picture of the social behaviour of Borsa Italiana equity traders. The volatility can be seen to drop off in the interval 12:30 - 14:00 and to grow suddenly again around 14:20. These times correspond to the typically preferred lunchtime interval of most traders. In Fig. 8(b), we plot the number of trades on the FTSE MIB index as a function of the interval index k for $\delta t = 300$ s. The behavior of the trade activity closely follows the behavior of volatility. This is even clearer from the analysis of Fig. 8(c) where the volatility is plotted as a function of the activity. The scatter plot shows a strong correlation between the two variables. This result does not depend on the length of the interval w , but the corresponding plots are not presented here for the sake of compactness. This feature was already present in the Australian market studied for a much longer period (10 years ≈ 2500 days) by [Bertram \(2004, 2005\)](#). Again, it is remarkable to see a statistical pattern still valid in a different market after more than 10 years.

Fig. 8 shows a clear seasonal pattern in intraday trades. In order to take this behavior into account, one of us proposed to use a non-stationary normal compound Poisson process with volatility of jumps proportional to the activity of the Poisson process in [Scalas \(2007\)](#). Here, we take even a more pragmatic stand and we do not assume any *a priori* relationship between volatility and activity as it emerges spontaneously, if present, with the method described in the next Section.

Empirical studies of volatility for daily financial data by [Bouchaud, Matacz and Potters](#)

(2001) have shown that volatility estimates and returns are negatively correlated for positive time lag. Therefore, following Bouchaud, Matacz and Potters (2001), we investigate this effect on high frequency data by estimating the leverage correlation function as

$$(4.2) \quad L(\Delta t) = \frac{\langle (r^I(t + \Delta t))^2 r^I(t) \rangle}{(\text{var}[r^I(t)])^2},$$

where Δt represents the lag. The estimates for empirical data samples are shown in Fig. 9. The leverage effect is not present.

5. Model. As one can see, during a trading day, the volatility and the activity are higher at the opening of the market, then they decrease at midday and they increase again towards market closure Bertram (2004) (see also Fig. 8). In other words, the (log-)price process is non-stationary. As suggested in Scalas (2007), such a non-stationary process for log-prices can be approximated by a mixture of normal compound Poisson processes (NCPP) in the following way. A normal compound Poisson process is a compound Poisson process with normal jumps. In formula, one can write:

$$(5.1) \quad X(t) = \sum_{i=1}^{N(t)} R_i,$$

where R_i are normally distributed independent trade-by-trade log-returns, $N(t)$ is a Poisson process with parameter λ and $X(t)$ is the logarithmic price, $X(t) = \log(P(t))$. By probabilistic arguments one can derive the cumulative distribution function of $X(t)$, it is given by:

$$(5.2) \quad F_{X(t)}(u) = \mathbb{P}(X(t) \leq u) = e^{-\lambda t} \sum_{n=0}^{\infty} \frac{(\lambda t)^n}{n!} F_R^{*n}(u),$$

where $F_R^{*n}(u)$ is the n -fold convolution of the normal distribution, namely

$$(5.3) \quad F_R^{*n}(u) = \frac{1}{2} \left[1 + \text{erf} \left(\frac{u - n\mu}{\sqrt{2n\sigma^2}} \right) \right],$$

and μ and σ^2 are the parameters of the normal distribution.

We now assume that the trading day can be divided into T equal intervals of constant activity $\{\lambda_i\}_{i=1}^T$ and of length w , then the unconditional waiting time distribution becomes a mixture of exponential distributions and its cumulative distribution function can be written as

$$(5.4) \quad F_\tau(u) = \mathbb{P}(\tau \leq u) = \sum_{i=1}^T a_i (1 - e^{-\lambda_i \tau}),$$

where $\{a_i\}_{i=1}^T$ is a set of suitable weights. The activity seasonality can be mimicked by values of μ_i that decrease towards midday and then increase again towards market closure. In order to reproduce the correlation between volatility and activity, one can assume that

$$(5.5) \quad \sigma_{\xi,i} = c\lambda_i$$

where c is a suitable constant. As already mentioned, however, for practical purposes, one can estimate three parameters for each interval, the parameter λ_i of the Poisson process and the parameters μ_i and σ_i for the log-returns. The estimates are:

$$(5.6) \quad \hat{\lambda}_i = N_i/w;$$

$$(5.7) \quad \hat{\mu}_i = \frac{1}{N_i} \sum_{k=1}^{N_i} r_i;$$

$$(5.8) \quad \hat{\sigma}_i^2 = \frac{1}{N_i - 1} \sum_{k=1}^{N_i} (r_i - \hat{\mu}_i)^2,$$

where N_i is the number of trades in the i th interval.

A Monte Carlo simulation of the model described above was performed by considering a trading day divided into a number of intervals of length $w = 3, 5, 10, 30, 300$ s. The parameters $\hat{\lambda}_i$, $\hat{\mu}_i$ and $\hat{\sigma}_i^2$ were estimated as explained above. In the following, we shall focus on estimates based on the FTSE

MIB index. Fig. 10 displays the histogram of simulated returns and can be compared to Fig. 6. In Fig. 11, we empirically show that the simulation gives a better fit for the empirical returns of the index as w vanishes. This is an encouraging result meaning that it will be useful to study the convergence of the approximation by means of measure-theoretical probabilistic methods. In order to show that this approximation is able to reproduce the stylized facts described above, Fig. 12 shows the scaling relations discussed in section 2.2 for the simulation with $w = 10$ s. One can see from Fig. 12(b) that an OLS index estimate $\hat{\alpha}_L = 1.59$ is recovered from the simulation instead of 1.72 for the real index. The scaling given in Eqs. (3.6), (3.7) is presented in Fig. 12(c), one can see that the approximate scaling still holds for the simulated data. In Fig. 13, we can see that there is no clear leverage effect in the simulated data as in the real case. Finally, in Fig. 14, for the simulated time series, we repeat the same analysis presented in Fig. 8. Given that, by construction, the non-stationary behavior of the simulated data is modeled on the non-stationary behavior of the real data, it is no surprise to find a good qualitative match between the two analyses (see Figs. 8,14).

6. Conclusions. In this paper, we addressed two questions. The first one concerns to so-called stylized facts for high-frequency financial data. In particular, do the statistical regularities detected in the past still hold? We can give a positive answer to this question. Indeed, we find that the scaling properties for financial returns are still approximately satisfied. Most of the studies we refer to concerned a different market (the US NYSE) and were performed several years ago. However, one of the first econophysics papers (if not the first one) concerned returns in the Italian stock exchange (see Mantegna (1991)) and, for this reason, we decided to focus on this market.

The second question is: Is it possible to approximate the non-stationary behaviour of intra-day tick-by-tick returns by means of a simple phenomenological stochastic process? Again, we can give a positive answer to this question. In Section 5, we present a simple non-homogeneous normal compound

Poisson process and we show that it can approximate empirical data. The cost for simplicity is over-fitting as we have to estimate many parameters, but the outcome is a rather accurate representation of the real process. Whether it is possible to rigorously prove convergence of the method outlined in Section 5 is subject to further research and it is outside the scope of the present paper. It is well-known that Lévy processes, namely stochastic processes with stationary and independent increments, can be approximated by compound Poisson processes. The method described in Section 5 can provide a clue for a generalization of such a result to processes with non-stationary and non-independent increments.

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TABLE 1

*Symbols and number of observations for the 40 assets composing the FTSE MIB index in
February-March 2011*

Asset	Symbol	Number of observations
A2A	A2A	17987
Ansaldo STS	STS	14252
Atlantia	ATL	25811
Autogrill Spa	AGL	15834
Azimut	AZM	14779
Banco Popolare	BP	70373
Bca MPS	BMPS	38005
Bca Pop Milano	PMI	32132
Bulgari	BUL	20164
Buzzi Unicem	BZU	25236
Campari	CPR	14789
Diasorin	DIA	16386
Enel	ENEL	73223
Enel Green Power	EGPW	29305
ENI	ENI	77280
Exor	EXO	26108
Fiat	F	84641
Fiat Industrial	FI	52212
Finmeccanica	FNC	31566
Fondiaria-SAI	FSA	21169
Generali Ass	G	60561
Impregilo	IPG	16414
Intesa Sanpaolo	ISP	84525
Lottomatica	LTO	14313
Luxottica Group	LUX	25717
Mediaset	MS	32019
Mediobanca	MB	37848
Mediolanum	MED	17185
Parmalat	PLT	30861
Pirelli & C	PC	27023
Prysmian	PRY	32806
Saipem	SPM	57592
Snam Rete Gas	SRG	25324
STMicroelectronics	STM	54515
Telecom Italia	TIT	49576
Tenaris	TEN	36410
Terna	TRN	21836
Tod's	TOD	14811
Ubi Banca	UBI	31541
UniCredit	UCG	168433
Index	FTSE MIB	405560

TABLE 2

Descriptive statistics for the waiting times τ^h

Asset	mean	std	α	β	AD
A2A	32.49	39.04	0.053	0.865	233
STS	34.07	43.68	0.061	0.818	283
ATL	24.42	32.48	0.088	0.792	376
AGL	33.20	41.87	0.059	0.830	336
AZM	34.67	42.35	0.052	0.853	291
BP	9.54	12.80	0.189	0.786	1345
BMPS	17.21	23.96	0.130	0.761	382
PMI	19.95	27.26	0.111	0.773	278
BUL	24.87	37.02	0.116	0.717	409
BZU	22.62	33.71	0.125	0.716	494
CPR	33.77	42.42	0.058	0.833	433
DIA	30.21	39.91	0.073	0.797	291
ENEL	9.19	11.60	0.173	0.829	1181
EGPW	21.16	29.31	0.110	0.764	228
ENI	8.71	12.21	0.221	0.756	1933
EXO	22.72	31.16	0.101	0.771	224
F	7.94	11.29	0.243	0.747	2533
FI	12.80	18.77	0.182	0.726	920
FNC	20.86	26.98	0.093	0.812	237
FSA	23.70	35.15	0.120	0.719	343
G	11.10	14.79	0.165	0.792	897
IPG	32.26	41.41	0.064	0.818	334
ISP	7.96	11.30	0.242	0.748	2509
LTO	33.22	42.54	0.062	0.819	261
LUX	23.28	31.52	0.096	0.780	259
MS	20.12	27.93	0.114	0.763	350
MB	17.40	24.03	0.126	0.767	379
MED	31.66	39.57	0.060	0.837	259
PLT	20.49	29.01	0.119	0.749	307
PC	22.78	30.45	0.094	0.789	235
PRY	19.48	27.87	0.126	0.743	375
SPM	11.53	17.88	0.219	0.691	1408
SRG	24.77	32.77	0.086	0.796	261
STM	12.22	17.26	0.174	0.751	891
TIT	13.27	20.52	0.198	0.692	1010
TEN	17.49	24.98	0.137	0.743	386
TRN	28.12	35.52	0.068	0.829	243
TOD	31.31	40.71	0.068	0.808	197
UBI	20.58	27.30	0.100	0.794	280
UCG	3.85	4.94	0.364	0.817	10855
Index	1.66	1.26	—	—	—

TABLE 3
Descriptive statistics for the volumes v^h

Assets	mean $\times 10^4$	variance $\times 10^8$	skewness	kurtosis $\times 10^2$
A2A	1.11	5.72	11.17	2.75
STS	0.11	0.05	10.86	2.79
ATL	0.16	0.09	8.79	2.16
AGL	0.15	0.09	7.97	1.26
AZM	0.13	0.05	6.10	0.70
BP	1.17	6.21	20.98	12.14
BMPS	1.69	10.05	6.98	1.01
PMI	0.52	0.67	5.64	0.74
BUL	0.53	7.33	26.99	13.21
BZU	0.16	0.07	7.05	1.21
CPR	0.18	0.08	5.66	0.61
DIA	0.03	0.28×10^{-2}	6.33	0.73
ENEL	1.09	7.06	15.92	6.97
EGPW	0.80	2.78	12.88	3.60
ENI	0.48	2.20	78.73	118.40
EXO	0.07	0.01	5.10	0.49
F	0.62	1.68	9.31	2.05
FI	0.31	0.37	8.04	1.36
FNC	0.18	0.14	10.76	3.01
FSA	0.22	0.14	10.39	3.42
G	0.31	0.35	9.09	2.32
IPG	0.56	1.40	13.44	3.88
ISP	3.39	45.25	7.56	1.72
LTO	0.14	0.07	6.67	0.81
LUX	0.08	0.02	10.30	2.83
MS	0.41	0.54	7.23	1.19
MB	0.28	0.26	8.41	1.66
MED	0.31	0.33	10.09	2.29
PLT	1.01	8.72	31.52	17.87
PC	0.33	0.37	9.07	2.07
PRY	0.14	0.07	7.80	1.32
SPM	0.09	0.03	13.07	5.58
SRG	0.56	6.92	117.04	166.34
STM	0.29	0.32	7.21	1.29
TIT	3.26	66.70	18.81	11.30
TEN	0.17	0.09	9.18	2.11
TRN	0.83	5.89	61.35	64.82
TOD	0.02	0.08×10^{-2}	7.52	1.07
UBI	0.23	0.17	6.87	1.01
UCG	5.63	124.95	7.82	1.78

TABLE 4

Descriptive statistics for the trade-by-trade log-returns r^h . () On March 7th, 2011, the French firm LVMH launched a takeover offer (OPA - Offerta Pubblica d'Acquisto in Italian) to buy Bulgari shares at 12.25 euros. On that day, this share price jumped from below 8 euros to more than 12 euros.*

Assets	mean $\times 10^{-7}$	variance $\times 10^{-7}$	skewness $\times 10^{-2}$	kurtosis
A2A	29.15	5.24	9.36	5.22
STS	-14.43	6.76	-7.11	11.50
ATL	1.59	2.09	24.62	19.64
AGL	-36.50	6.09	114.90	43.47
AZM	-3.29	8.03	-21.90	14.14
BP	-4.53	4.55	-1.69	10.69
BMPS	24.93	4.79	-21.71	24.34
PMI	6.87	5.55	-23.73	41.72
BUL (*)	-3.75	4.37	-295.68	154.69
BZU	61.92	7.41	-99.04	35.92
CPR	2.35	3.73	11.04	8.13
DIA	-40.04	4.42	-49.99	29.17
ENEL	6.21	1.38	140.10	76.06
EGPW	38.81	3.64	3.43	7.31
ENI	7.86	1.40	59.89	21.01
EXO	11.98	4.82	-5.45	8.06
F	-3.55	2.81	-45.05	21.76
FI	14.33	3.68	-39.37	18.14
FNC	0.50	3.29	28.01	13.01
FSA	84.68	10.35	-163.51	180.64
G	5.03	2.09	-100.65	44.97
IPG	80.66	9.04	-45.81	22.68
ISP	1.99	3.45	-62.87	43.12
LTO	67.82	9.28	-171.44	62.62
LUX	25.88	2.67	30.48	24.43
MS	5.76	2.86	-22.98	19.38
MB	17.29	4.18	1.66	9.67
MED	20.25	7.64	-43.78	18.78
PLT	9.76	5.30	49.56	14.43
PC	47.93	5.41	3.44	10.75
PRY	21.54	4.02	257.09	92.76
SPM	5.72	1.50	-9.12	32.75
SRG	12.09	2.41	79.03	54.87
STM	15.69	2.56	-39.64	36.78
TIT	8.33	3.20	-22.22	8.92
TEN	0.34	2.61	-112.99	135.05
TRN	26.67	2.42	3.54	6.03
TOD	28.73	6.95	158.96	86.49
UBI	-1.76	4.99	-67.53	25.23
UCG	3.44	1.29	-12.56	57.51
Index	1.10	0.03	2	8.54

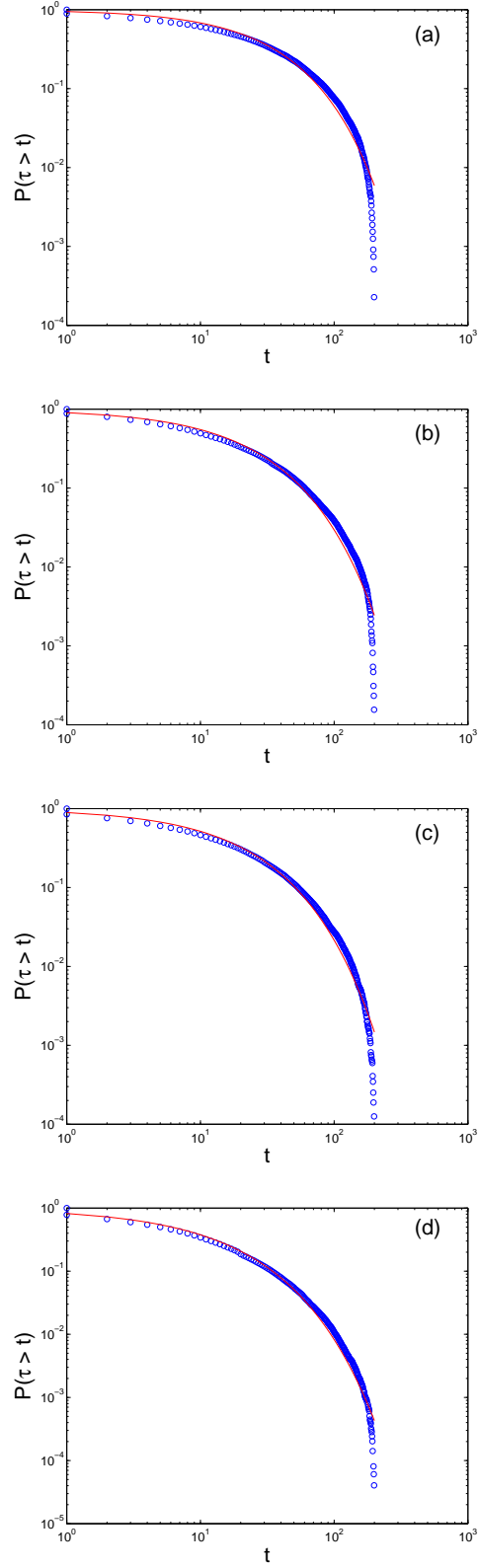


FIG 1. Weibull fit for A2A (a), EXO (b), MS (c), TIT (d). The fit is represented by the thin solid line, the open circles are the empirical values for the survival function.

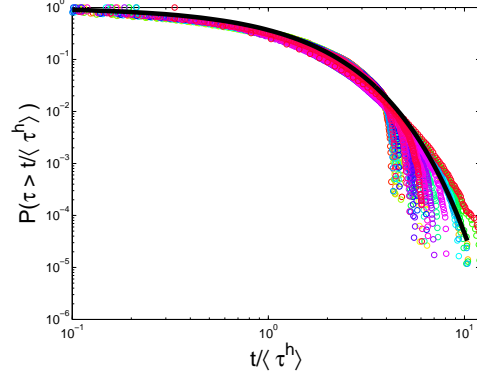


FIG 2. (Color online) Approximate scaling of the survival function for the forty time series. The solid line is the Weibull fit given by Eq.(3.3).

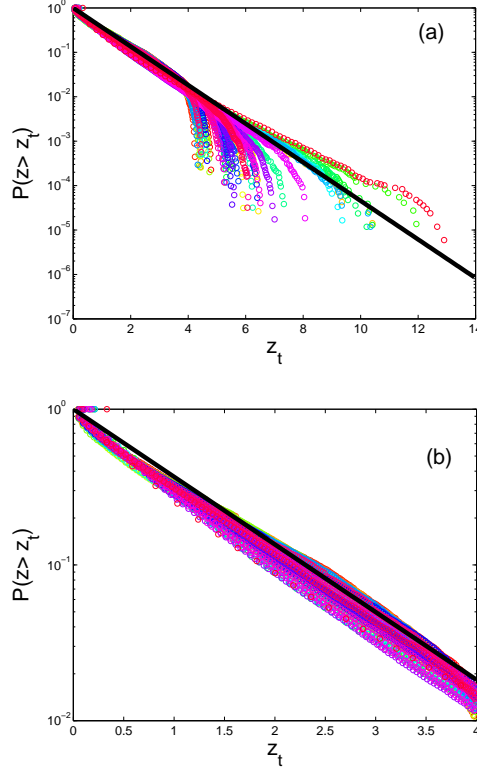


FIG 3. (Color online) (a) Empirical survival function for the transformed variable z_t^h compared with the expected exponential function $\exp(-z_t^h)$; (b) Zoom in the region $z_t^h \leq 4$.

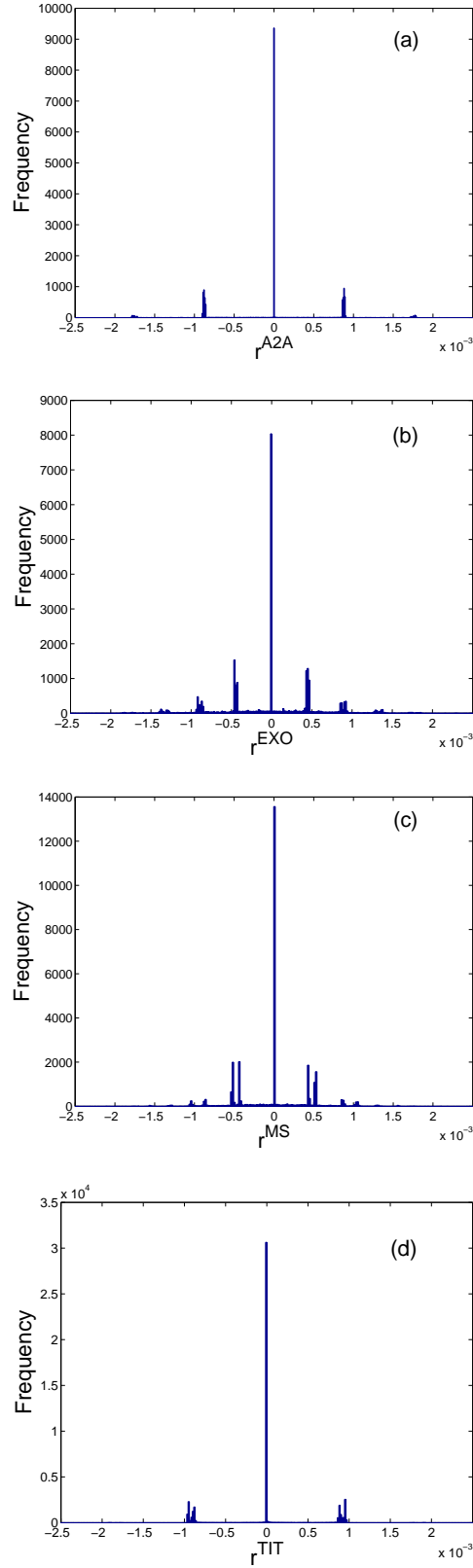


FIG 4. (Color online) Histogram of returns for A2A (a), EXO (b), MS (c), TIT (d).

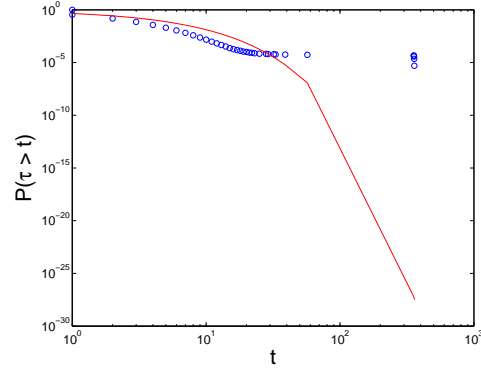


FIG 5. (Color online) Circles: empirical survival function; solid line: Weibull fit.

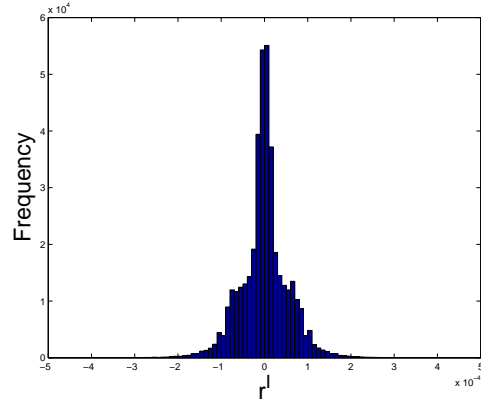


FIG 6. (Color online) Histogram of returns for the FTSE MIB index.

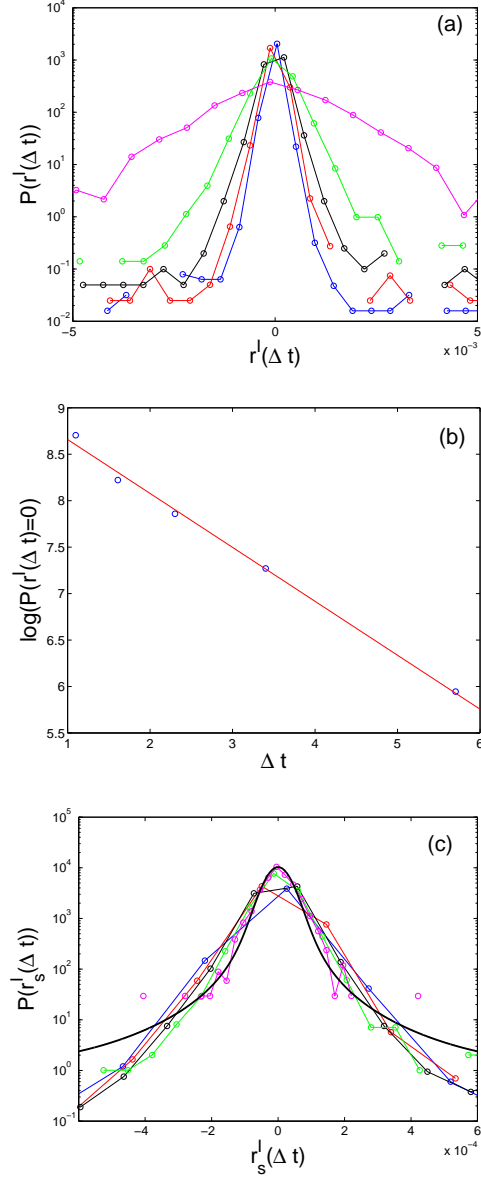


FIG 7. (Color online) (a) Histogram of the returns for the FTSE MIB index observed at different time intervals, namely, $\Delta t = 3$ s (blue), 5 s (red), 10 s (black), 30 s (green) and 300 s (purple); (b) Probability of zero returns as a function of the time sampling interval Δt , the slope of the straight line is 0.58 ± 0.01 ; (c) scaled empirical probability distribution and comparison with the theoretical prediction given by Eq.(3.7) (black solid line).

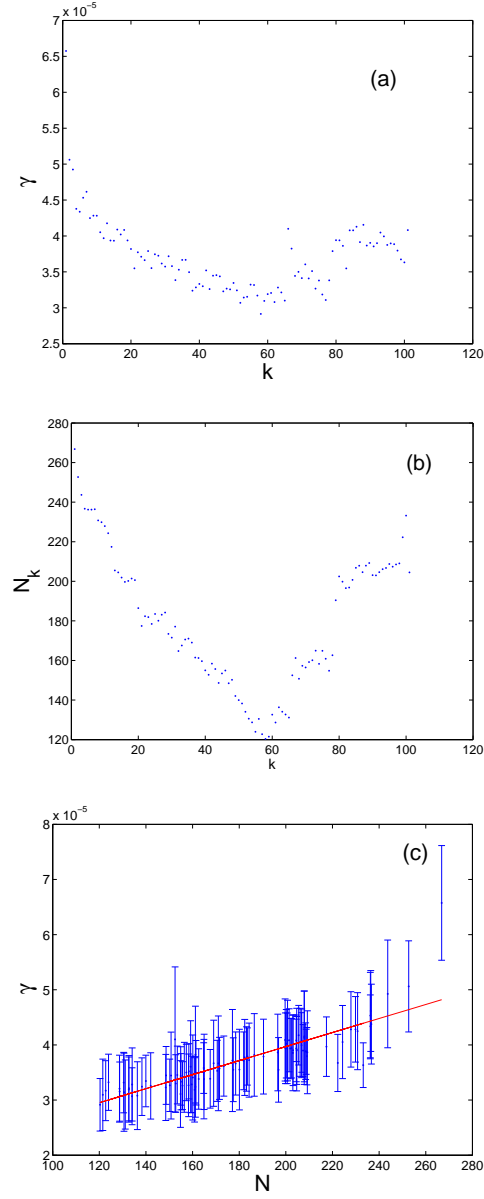


FIG 8. (a) Volatility γ as a function of k for $\delta t = 300$ s. (b) Activity N as a function of k for $\delta t = 300$ s. (c) Scatter plot of volatility γ as a function of number of trades N . The points are averaged over the investigated period.

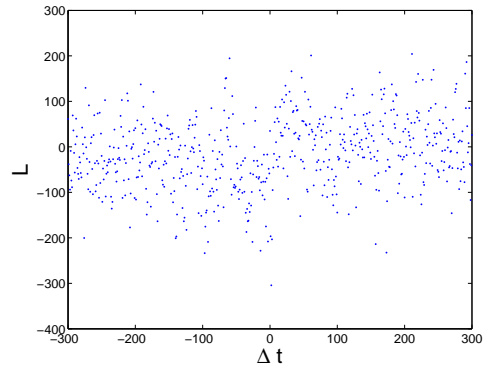


FIG 9. Leverage L as a function of lag Δt . There is no strong evidence of leverage effect.

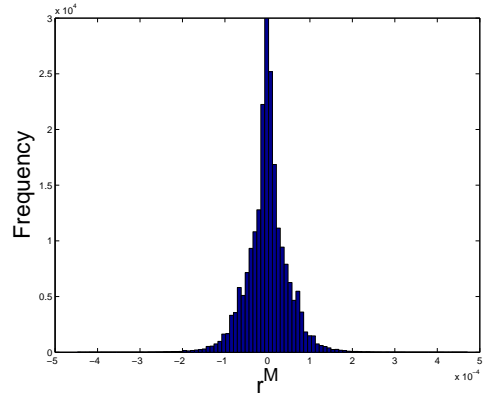


FIG 10. (Color online) Histogram of returns for the approximating process with $w = 3s$.

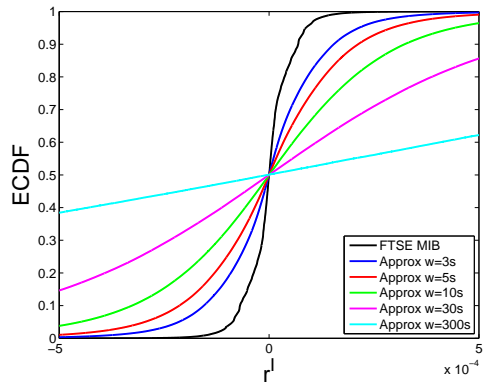


FIG 11. (Color online) Approximation of the empirical cumulative distribution function for FTSE MIB returns r^I .

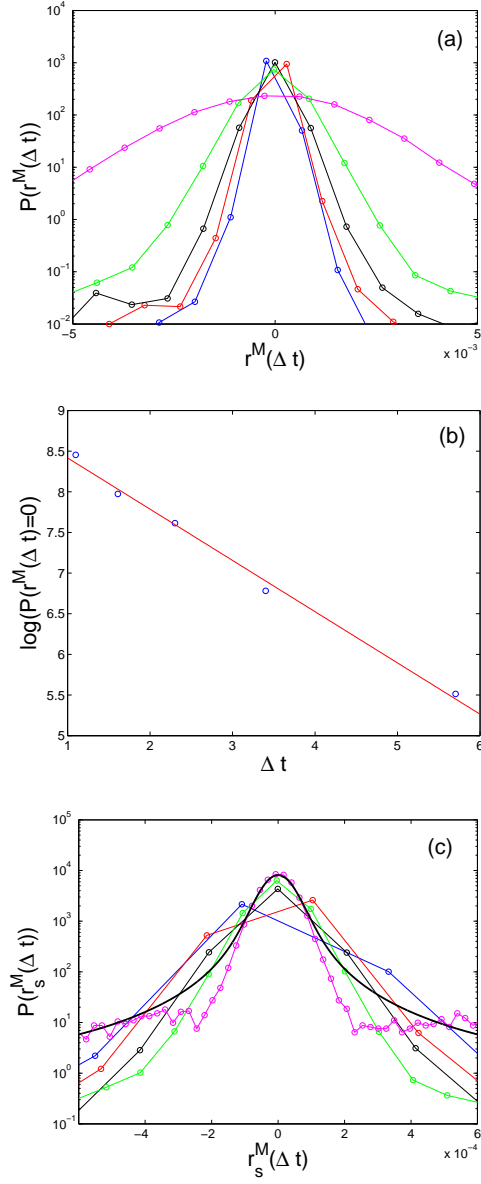


FIG 12. (Color online) (a) Histogram of the returns for the simulation described in the text observed at different time intervals, namely, $\Delta t = 3$ s (blue), 5 s (red), 10 s (black), 30 s (green) and 300 s (purple); (b) Probability of zero returns as a function of the time sampling interval Δt , the slope of the straight line is 0.63 ± 0.01 ; (c) scaled empirical probability distribution and comparison with the theoretical prediction given by Eq.(3.7) (black solid line).

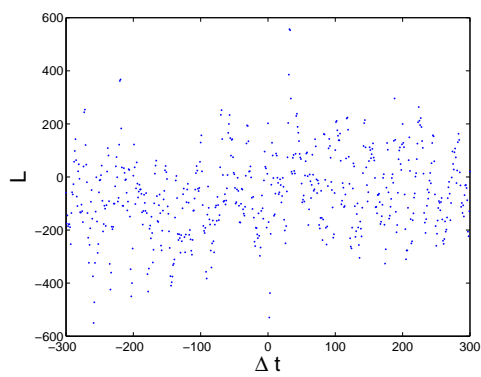


FIG 13. (Color online) Leverage L as a function of lag Δt for simulated data. Also for the simulation there is no evidence of leverage effect.

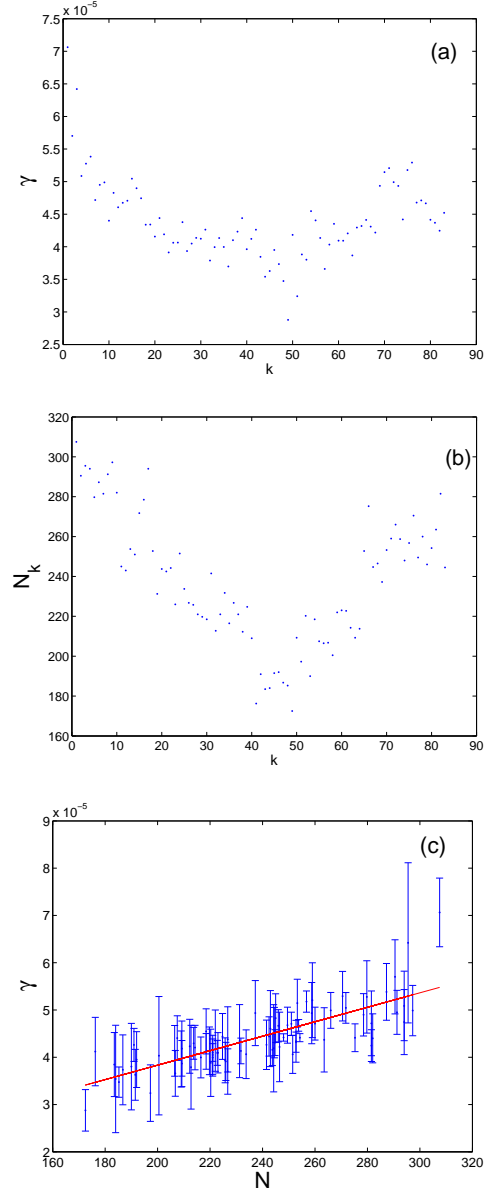


FIG 14. (Color online) (a) Volatility γ as a function of k for $\delta t = 300$ s. (b) Activity N as a function of k for $\delta t = 300$ s. (c) Scatter plot of volatility γ as a function of number of trades N . The points are averaged over the investigated period. All the plots are for simulated data with $w = 10$ s.